

## GLOBAL JOURNAL OF ENGINEERING SCIENCE AND RESEARCHES EXPLAINING THE RELATION OF NEUTRON ENERGY SCATTERING CROSS SECTION INTENSITY YIELD USING GENERALIZED MAXWELL'S STATISTICAL EQUATION

Mubarak Dirar Abdallah<sup>\*1</sup>, Mohammed Ezzeldin Elhibir<sup>2</sup>, Ahmed Elhassen Alfaki<sup>3</sup> & Sawsan Ahmed Elhoury Ahmed<sup>4</sup>

<sup>\*1</sup>Sudan University of Science & Technology-College of Science-Department of Physics- Khartoum-Sudan & International University of Africa- College of Science-Department of Physics Khartoum-Sudan

<sup>2&3</sup>Sudan University of Science & Technology-College of Science-Department of Physics- Khartoum-Sudan

<sup>4</sup>University of Bahri-College of Applied & Industrial Sciences-Department of Physics-Khartoum - Sudan

### ABSTRACT

The relation of the paper title is explained by assuming neutrons interacting magnetically with the target. The theoretical relations are adequately explaining all empirical relations.

**Keywords:** scattering cross section, yield, Intensity, Energy, flux, generalized Maxwell' equation.

### I. INTRODUCTION

Atoms are building blocks of matter any atom consists of three elementary particles, which are neutrons, protons and electrons[1]. The protons and electrons have positive and negative charges respectively; while neutrons are electrically neutral[2]. However neutrons have magnetic moments. This striking feature makes them capable of probing and able to determine the magnetic properties of matter. Neutrons are used widely in technology. It is used in nuclear fusion process which enables generation of electric energy in nuclear power stations[3,4]. It is also used in determining elements existing in rocks, rocks using neutron activation spectrometer. Neutron scattering Process is widely used in the experiments of elementary particles[5,6]. It is used for discovering the particles and mediators predicted by unification models [7,8,9]. The neutron scattering process cannot be easily explained by using simple physical laws [10]. This motivates trying to use generalized statistical physics to explain some scattering processes.

### II. THE SCATTERING GENERALIZED MAXWELL DISTRIBUTION MODEL

According to the generalized statistical laws (GSL) the Cross section  $\sigma$  and the intensity  $I$  are proportional to the incident flux or scattered flux  $n$  according to the relations[1].

$$\sigma \sim n \sim n_0 e^{-\frac{E}{E_0}} \quad (1)$$

$$I \sim n \sim n_0 e^{-\frac{E}{E_0}} \quad (2)$$

In the work done by Hadi Dawyic [1] the relation between Neutron energy  $E_n$  and the interaction cross section  $\sigma$  for  $^{231}\text{Th}$ ,  $^{233}\text{Th}$ ,  $^{239}\text{U}$  was investigated and the result is shown in table (1). This relation is displayed graphically in figure (1)

Table (1): Empirical relation between Neutron energy  $E_n$  and cross section  $\sigma_c$  (b) for same elements

$E_n$ (MeV)	Th <sup>231</sup>	Th <sup>233</sup>	U <sup>235</sup>	U <sup>237</sup>	U <sup>239</sup>	Pu <sup>241</sup>	Pu <sup>243</sup>
	$\sigma_c$ (b)	$\sigma_c$ (b)	$\sigma_c$ (b)	$\sigma_c$ (b)	$\sigma_c$ (b)	$\sigma_c$ (b)	$\sigma_c$ (b)
0.5	3.65	3.65	3.65	3.65	3.66	3.66	3.67
0.6	3.62	3.63	3.63	3.64	3.64	3.64	3.65
0.7	3.61	3.62	3.62	3.63	3.63	3.63	3.64
0.8	3.62	3.63	3.63	3.63	3.64	3.63	3.65
1.0	3.63	3.64	3.63	3.64	3.65	3.65	3.66
1.25	3.78	3.69	3.67	3.67	3.68	3.69	3.69
1.5	3.72	3.73	3.70	3.70	3.71	3.72	3.72
1.75	3.73	3.74	3.70	3.70	3.72	3.72	3.72
2	3.75	3.76	3.71	3.70	3.72	3.72	3.72

The empirical relation between incident neutron energy  $E_n$  and Interaction cross section  $\sigma_c$  (b) is displayed graphically in figure (1)

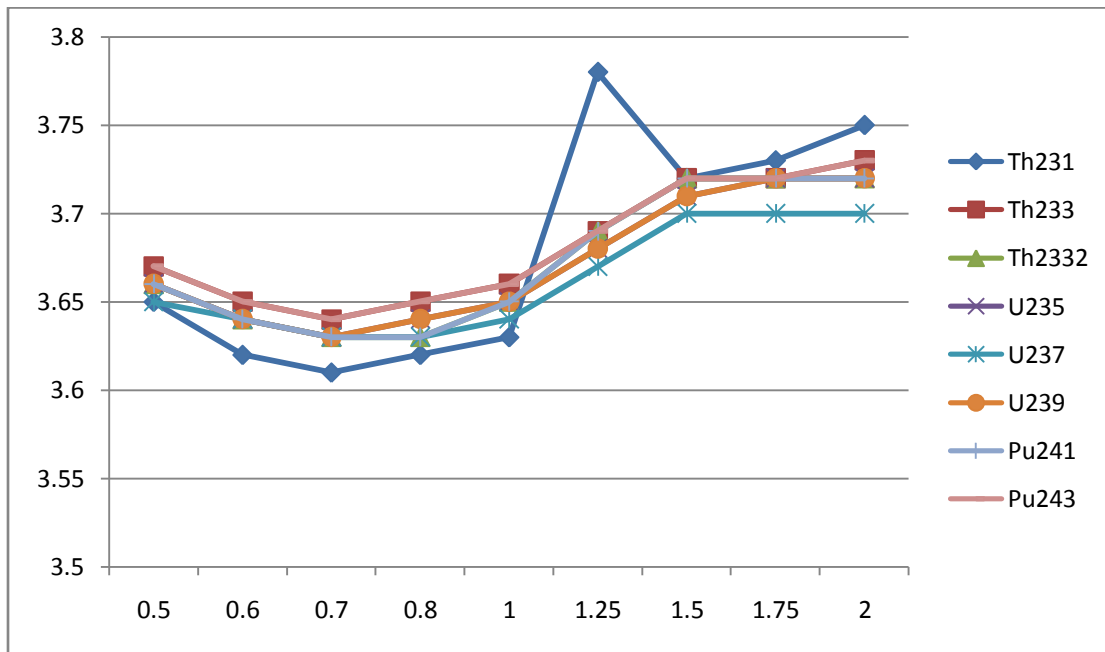


Figure (1) empirical relation of  $E_n$  versus  $\sigma_c$  (b) for different element

The empirical relation can be explained by using the generalized Maxwell distribution (GMD) where the average energy can be assumed to result mainly from the strong magnetic interaction between the neutron magnetic moment and the magnetic field of the target nuclei. if one assumes that this interaction potential ( $V_0$ ) is strong and attractive such that the kinetic energy can be neglected. In this case the average energy is given by

$$E = -V_0 \quad (3)$$

Thus the number of scattered neutrons is given by

$$n = n_0 e^{\frac{E_n}{V_0}} \quad (4)$$

Where the scattered neutron energy is given by

$$E_n = \gamma_o E \quad (5)$$

By assuming that part of incident neuron energy  $E$  goes to the scattered one, thus

$$n = n_0 e^{\frac{\gamma_0 E}{V_0}} \quad (6)$$

But the intensity  $I$  is given by

$$I = nv \quad (7)$$

$$I = I_0 e^{\frac{\gamma_0 E}{V_0}} \quad (8)$$

Where

$$I_0 = n_0 v \quad (9)$$

Thus the scattering cross section  $\sigma$  is given by

$$\sigma = N_{sc} \sim I = I_0 e^{\frac{\gamma_0 E}{V_0}} \quad (10)$$

For simplicity, let

$$V_0 \sim 1$$

$$\gamma_0 \sim 1$$

Therefore

$$\sigma = I_0 e^E \quad (11)$$

Further simplification yields

$$I_0 = 10$$

To get

$$\sigma = 10e^E \quad (11 - 1)$$

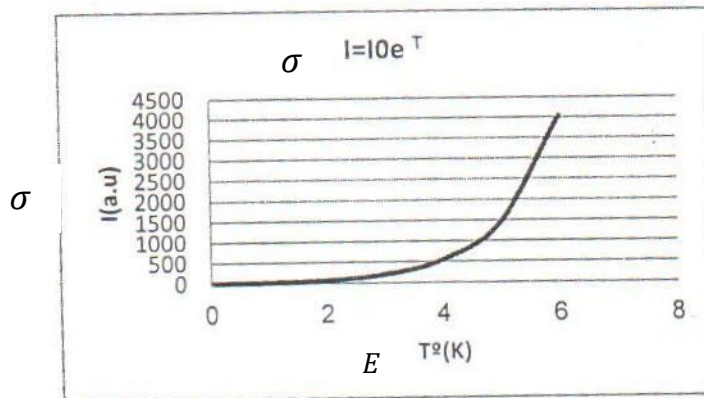


Fig (2) Theoretical relation between  $E$  and  $\sigma$

The theoretical relation between  $E$  and  $I$ , in equation (8) can also be used to explain proton – neutron scattering. One can simplify equation for intensity  $I$  in equation (8), i.e.

$$I = I_0 e^{\frac{\gamma_0 E}{V_0}} \quad (12)$$

By setting

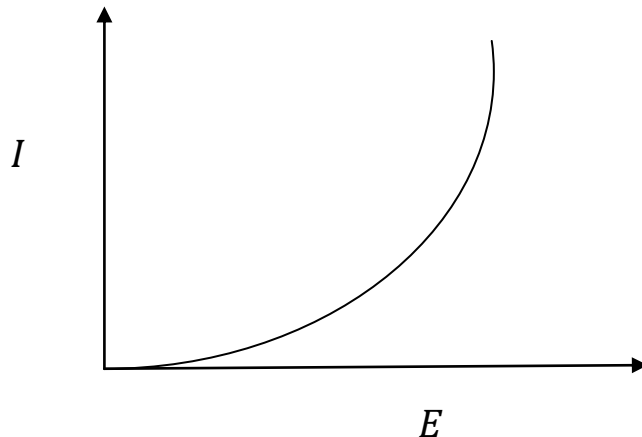
$$I_0 \sim 10$$

And

$$\frac{\gamma_0}{V_0} \sim 1$$

To get

$$I = 10e^E \quad (12.1)$$



*Fig (3) Theoretical relation between E and I*

This theoretical relation can explain the experimental relation between I and E for the scattering process  ${}^9\text{Be}(p,n){}^9\text{B}$  which is shown in table (2)

*Table (2): Empirical relation between n and I for  ${}^9\text{Be}(p,n){}^9\text{B}$*

n	I	E (Mev)
1.6e+6	2.58e-7	2.10
8.06e+6	1.29e-6	2.20
5.71e+7	9.13e-6	2.50
2.65e+8	4.23e-5	3.0
5.19e+8	8.30e-5	3.50
9.82e+8	1.57e-4	4.0
1.49e+9	2.38e-4	4.50
2.25e+9	3.60e-4	5.0
4.34e+9	6.95e-4	6.0
5.43e+9	8.69e-4	6.50
7.11e+9	1.14e-3	7.0

It is very interesting to note that the theoretical curve of I versus E in fig (3) resembles the empirical one in fig (4)

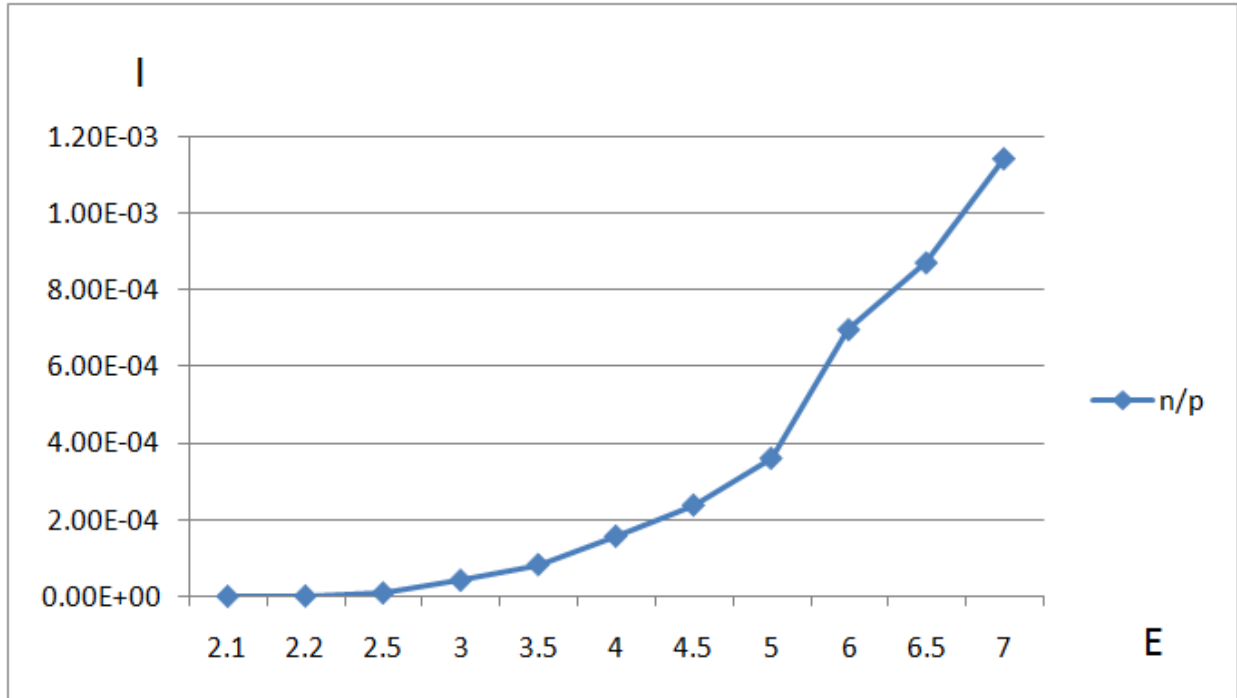


Fig (4) Empirical relation between I and E for <sup>9</sup>Be(p,n)<sup>9</sup>B scattering

The theoretical relation between yield and energy can be found by using the relation

$$y \sim n \sim e^{\frac{-E}{E}} \quad (13)$$

Hence

$$y \sim n_0 e^{\frac{-E}{E}}$$

$$y = y_0 e^{\frac{E}{E_0}} \quad (14)$$

If the neutrons which have magnetic moment are affected by attractive magnetic force the average energy becomes

$$\bar{E} = E_0 = -V_0 \quad (15)$$

Hence inserting this relation in (14) requires

$$y = y_0 e^{\frac{E}{V_0}} \quad (16)$$

Equation (16) can be displayed graphic in figure (5)

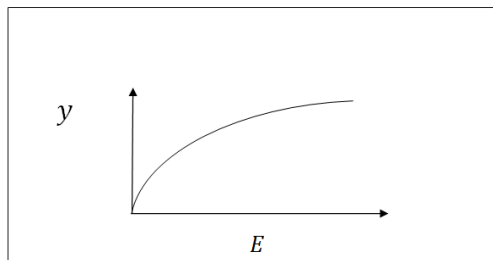


Figure (5) theoretical relation between y and E

It is very interesting to note that this theoretical relation in figure (5) can easily explain the empirical relation for  ${}^9\text{Be}(p,n){}^9\text{B}$  scattering which is displayed in figure (6)

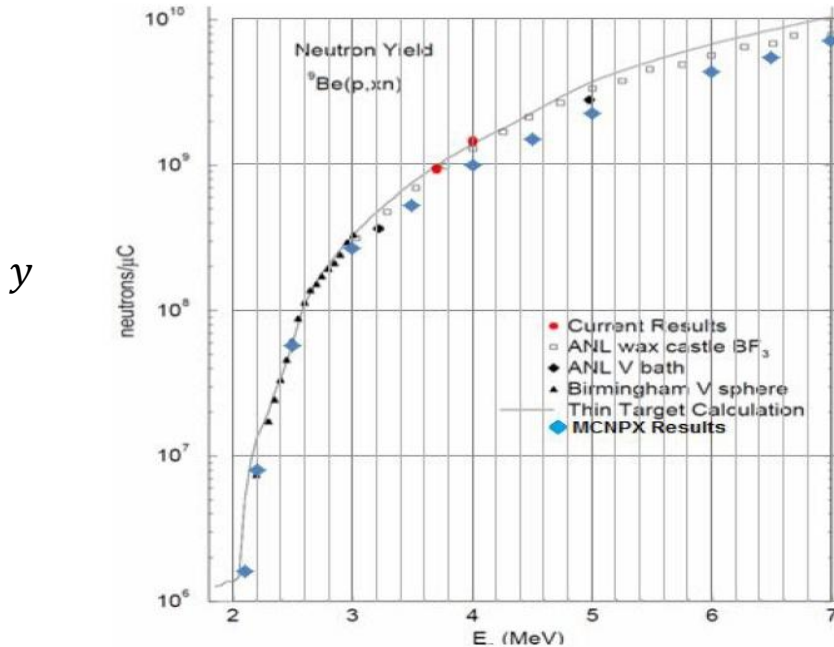


Figure (6) empirical relation between  $y$  and  $E$  for  ${}^9\text{Be}(p,n){}^9\text{B}$  scattering

The scattering Cross section  $\sigma$  for neutrons was shown to be related to the mass number  $A$ . The theoretical foundation can be made by assuming the neutron energy to result from interaction between the incident neutron having magnetic moment  $\mu_0$  and the atomic nucleus which have  $A$  protons and  $n$  neutrons. One can assume that each nucleon (proton or neutron) have net magnetic flux density  $B_0$ , i.e the proton and neutron have the same flux density  $B_0$ . by assuming further than all nucleon  $B_0$  point in the same direction thus the interaction energy between the nuclear and the neutron is given by

$$E_n = \mu_0(B_0) = \mu_0(B_0 A) = \mu_0 B_0 A \quad (17)$$

Where the nucleus net flux density is given by

$$B = B_0 A$$

Assuming the average interaction energy to be attractive thus according to equation (1)

$$\sigma = \sigma_0 e^{\frac{E}{V_0}} = \sigma_0 e^{\frac{\mu_0 B_0 A}{V_0}} \quad (18)$$

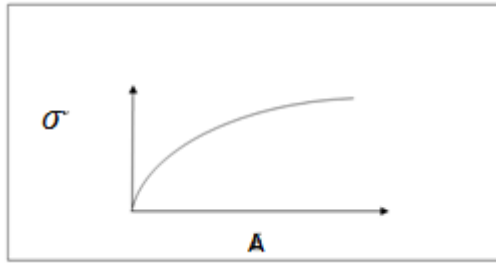


Figure (7) theoretical relation between  $\sigma$  and  $A$

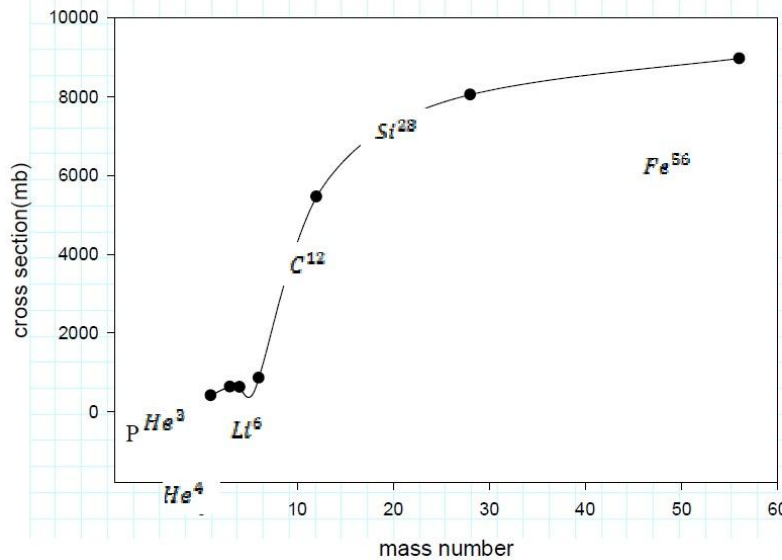


Figure (8) cross section  $\sigma$  against mass number  $A$

This theoretical relation shown in figure (7) conforms with the empirical one in figure (8). This empirical relation was found in the neutron scattering experiment done by Fatima Abu Zaghia and Mustafa Bayio, Jordon [3]

### III. DISCUSSION

Generalized Maxwell distribution laws shown in equation (1) is found to be successful in describing some scattering processes of neutrons . In all theoretical relations the neutrons are assumed to have magnetic moments and thus interact magnetically with the target. In view of figures (1) and (2) the relation between interaction cross section  $\sigma$  of neutrons for Th, U and Pu is explained by (GMD) assuming strong magnetic attraction and neglecting kinetic energy contribution or one can assume neutrons as vibrating string. Such that the average kinetic and potential terms are equal so that

$$T_o = V_o = \frac{m\omega^2 A^2}{2\sqrt{2}}$$

Thus

$$E = -2V_o$$

Then readjusting  $\gamma_o$  in equation (10) to be

$$\gamma_o = 2$$

One get the same result the scattering process  ${}^9\text{Be}(p,n){}^9\text{B}$  which is displayed empirically by GMD assuming again attractive string magnetic interaction which lead. To theoretical relation between  $E$  and  $I$  displayed in fig (3). This relation resembles the empirical one in fig (4). In the scattering process  ${}^9\text{Be}(p,n){}^9\text{B}$  the empirical relation between yield  $y$  and energy  $E$  can again be explained by GMD assuming average strong attractive potential as shown theoretically in fig (5). Finally GMD law can explain the relation between cross section  $\sigma$  and mass number (see fig (7)) by assuming magnetic interaction between neutrons and nuclei which have all magnetic moment of protons and neutrons point in the same direction, this theoretical relation resembles that of Fatima et al, made for  $\text{He}^4$ .

#### IV. CONCLUSION

Generalized Maxwell's statistical distribution law adequately explains relations of scattering cross section intensity and yield with neutron energy.

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